# QUANTUM CHEMICAL CALCULATIONS <br> ON 4-AZATRICYCLO[4,4,0,0 $\left.{ }^{3,8}\right]$ DECAN-5-ONE - A LACTAM WITH A NON-PLANAR cis-AMIDE GROUP 

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#### Abstract

The position of the $\mathrm{N}-\mathrm{H}$ bond in the lactam $I$ given in the title was optimized by the CNDO/2 method. On the basis of the found non-planarity of the cis-amide group ( $\Delta \omega=-14 \cdot 5^{\circ}, \theta_{\mathrm{N}}=26.4^{\circ}$, regular pyramid on the nitrogen atom), the values of oscillator and rotatory strengths of the $n-\pi^{*}$ and $\pi-\pi^{*}$ transitions were computed using the CNDO/S procedure. With the exception of the band positions, very good agreement with experimental results was obtained.


Recently the optical rotatory strengths of cyclic chiral amides have been investigated ${ }^{1-6}$ by quantum chemistry. These studies were carried out in order to recognize the behaviour of the amide chromophore in simple chiral systems, as well as to establish a basis for precise interpretation of the relationship between conformation and chiroptical properties of peptides and proteins. Optical activity of the $n-\pi^{*}$ transition of the planar amide group in N -acetylamino sugars and in chiral $\gamma$-lactams was investigated within the framework of one-electron theory ${ }^{3,7}$ and a quadrant rule indicating $C_{2 v}$ effective symmetry of this transition ${ }^{2}$ was suggested. According to semiempirical calculations of Ramachandran, Sasisekharan and coworkers ${ }^{8,9}$ compared with X-ray data ${ }^{9,10}$ the amide group can easily become non-planar due to torsion on the $\mathrm{C}-\mathrm{N}$ bond associated with a partial change of nitrogen atom hybridization, giving rise to a pyramidal arrangement. We should, therefore, relate parameters $\Delta \omega, \theta_{\mathrm{N}}$, characterizing the extent of non-planarity (Fig. 1), to the optical rotatory strength of both the $n-\pi^{*}$ and $\pi-\pi^{*}$ transitions of the amide group. Semiempirical calculations of optical rotatory strengths of several $\gamma$-lactams possessing non-planar amide groups were carried out ${ }^{5,6}$ using INDO CI and $\mathrm{CNDO} / \mathrm{S}$ wavefunctions. However, a systematic study has yet to be reported.

In a previous study ${ }^{11}$ we reported a set of $C D$ data on $(-)-(2 S, 5 S)-4$-azatricyclo-$-\left[4,4,0,0^{3,8}\right]$ decan-5-one ( $I$ ), a rigid lactam with a non-planar amide group. Upon separating the single Gaussian bands from the experimental CD curves of this compound, two significant Cotton effects were found within the range of $210-230 \mathrm{~nm}$ and at 195 nm respectively, corresponding to $n-\pi^{*}$ and $\pi-\pi^{*}$ transitions of the
amide group. In addition to these Cotton effects, a weak band was observed at $210-215 \mathrm{~nm}$ and tentatively assigned to a Rydberg $n-3 s$ transition in accord with published reports ${ }^{12,13}$. Experimental values of optical rotatory strength thus obtained are, particularly in the case of the $n-\pi^{*}$ transition, greater (an order of $10^{-39}$ cgs is reached) than the corresponding values related to chiral amides possessing planar amide groups. Considering the relatively high accuracy of these experimental data and the rigid structure of the lactam $I$ of known geometry in the crystalline state ${ }^{14}$, this compound appears suitable for performing a quantum chemical study of the optical rotatory strength of the non-planar amide group.

## CALCULATIONS

The preferred geometry of the amide group in the lactam (-)-I was studied by the standard CNDO/2 method, using the original parameterization ${ }^{15,16}$. Convergence of all orbital energies to $10^{-4} \mathrm{eV}$ was required for the SCF procedure to be terminated. As a first approximation of the Fock matrix, that resulting from the previous calculation was used. This procedure reduces by approximately one third the number of iterations required.


Due to the inherent chirality of the non-planar amide group, the optical rotatory strength should be calculated in a direct procedure, starting from the wave function obtained in the all valence electron framework. We have chosen the CNDO/S method suggested for the calculation of electronic spectra ${ }^{17}$. Repulsion integrals were approximated according to Mataga and Nishimoto ${ }^{18}$ and the parameterization used was taken from the report of Jaffé and coworkers ${ }^{19}$. A constant width of configuration interaction involving 36 singly excited configurations among six highest occupied and six lowest virtual orbitals was maintained in the whole series of calculations.


Fig. 1
The Angles $\Delta \omega, \Theta_{N}$ and $\Theta_{\mathrm{C}}$ Defined for the Case of the cis-Amide Group

The method of calculating the optical rotatory strength was derived from the direct procedure described by Gould and Hoffmann ${ }^{20}$. The original computer program was modified in order to enable utilization of the configuration interaction technique. Describing the wave function of the excited state $\psi_{\mathrm{j}}$ by a linear combination of Slater determinants representing the wave functions $\psi_{r}, \psi_{s}$ of the particular singly excited configurations, and expressing the matrix elements of electric and magnetic dipole operators in atomic units, the optical rotatory strength (in cgs units) amounts to:

$$
\begin{equation*}
R_{0 \mathrm{j}}=-6.412 .10^{-37} \sum_{\mathrm{r}, \mathrm{~s}} C_{\psi_{\mathrm{r}}} C_{\psi_{\mathrm{s}}} \frac{\left\langle\Psi_{0}\right| \nabla\left|\Psi_{\mathrm{r}}\right\rangle}{E_{\mathrm{r}}-E_{0}}\left\langle\Psi_{\mathrm{s}}\right| \mathbf{r} \times \nabla\left|\Psi_{0}\right\rangle \tag{1}
\end{equation*}
$$

where $C_{\psi \mathrm{r}}, C_{\psi \mathrm{s}}$ are the corresponding LCI coefficients and ( $E_{\mathrm{r}}-E_{0}$ ) represents the diagonal element of the LCI matrix given in eV . Thus, separately for each configuration, the electric transition moment calculated in gradient representation is transformed into a dipole length formalism. At least in the case of compound $I$, this procedure improves agreement of the computed values of rotatory strength with the experimental data. Although the transition moments were calculated with inclusion of all two-center integrals, the CNDO/S molecular orbitals used were not deorthogonalized ${ }^{21}$, since use of such a procedure does not influence the results qualitatively ${ }^{4}$. Numerical calculations were carried out by a CDC 3300 computer.

## RESULTS

Calculating the optical rotatory strength of the lactam (-)-I with the aid of the CNDO/S method, and starting with the geometry derived from X-ray data ${ }^{14}$, the

## Table I

CNDO/S Calculation of the Optical Rotatory Strength of Lactam (-)-I
The molecular geometry used was derived from the approximate geometry of the twistane skeleton ${ }^{22}$ on replacing the $\mathrm{C}_{(4)}$ atom by a nitrogen, then by addition of coordinates of hydrogens and of the oxygen atom of the amide group. The absolute configuration had to be reversed. The folowing supplementary geometrical parameters were used: Bond lengths $\mathrm{C}-\mathrm{H} 0.109 \mathrm{~nm}$, $\mathrm{C}=\mathrm{O} 0.124 \mathrm{~nm}, \mathrm{~N}-\mathrm{H} 0.102 \mathrm{~nm}$. A regular tetrahedral arrangement of the hydrogen atoms on skeletal carbons was assumed to occur. The torsion $\Delta \omega$ amounts to $-29^{\circ}$, the value of $\Theta_{\mathrm{N}}$ is $29^{\circ}$ (the $\mathrm{N}-\mathrm{H}$ bond eclipses the $\mathrm{C}=\mathrm{O}$ one).

| Quantity | $n-\pi^{*}$ transition | $\pi-\pi^{*}$ transition |
| :--- | :---: | :---: |
| $-\lambda_{\text {exp. }}{ }^{a}, \mathrm{~nm}$ | 232.0 | 197.0 |
| $\lambda_{\text {calc. }}, \mathrm{nm}$ | 280.4 | 178.9 |
| $R_{\text {exp. }}{ }^{a} \cdot 10^{39}, \mathrm{cgs}$ | -2.51 | 2.07 |
| $R_{\text {calc. }} \cdot 10^{39}, \mathrm{cgs}$ | -1.15 | 1.00 |

${ }^{a}$ Experimental values are related to a measurement performed in the cyclohexane solution ${ }^{11}$.
computed positions of both the $n-\pi^{*}$ and $\pi-\pi^{*}$ transitions, as well as the numerical values of rotatory strength, are reproduced very inaccurately ${ }^{11}$. On the other hand, the calculation based on the very approximate geometry deduced from the twistane skeleton ${ }^{22}$ (Table I) resulted in an improvement of the optical rotatory strength of the $\pi-\pi^{*}$ transition and of the positions of both bands in question. It appears clear from this finding that even with such a rigid compound the geometrical arrangement found in the crystalline state need not necessarily reproduce all the features of molecular geometry occurring in solution and that a geometry optimization can probably improve the results. The sole atom which may substantially alter its position in solution is the hydrogen atom bonded to the nitrogen atom of the amide group. Calculating its optimum position we start from: a) the Ramachandran relation

$$
\begin{equation*}
\Theta_{N}=-2 \Delta \omega \tag{2}
\end{equation*}
$$

which is not fulfilled in the case of crystallographic data on lactam $I\left(\Delta \omega=-14.49^{\circ}\right.$, $\Theta_{\mathrm{N}}=14.07^{\circ}$ ). The angle $\Theta_{\mathrm{N}}$ determined by the position of the above hydrogen atom, and representing the extent of the pyramidal character of the bonds attached to nitrogen, is too small; $b$ ) the $\mathrm{N}-\mathrm{H}$ bond length $(0.0919 \mathrm{~nm})$ is small in comparison with the recommended average value ${ }^{23}(0.1 \mathrm{~nm})$. The position of the hydrogen atom


Fig. 2
The Total CNDO/2 Energy of the Lactam (-)-I Related to the Angle $\Theta_{\mathrm{N}}$

Computed for the $\mathrm{N}-\mathrm{H}$ bond length: $10.0919 \mathrm{~nm}, 20.1065 \mathrm{~nm}$. A value of $E=$ $=0 \mathrm{kcal} \mathrm{mol}^{-1}$ was arbitrarily assigned to the structure exhibiting minimum energy.


Fig. 3
The CNDO/2 Energy of the Lactam (-)-I Related to the $\mathrm{N}-\mathrm{H}$ Bond Length

The calculation was performed for $\Theta_{N}=$ $=21 \cdot 4^{\circ}$. The energy scale is related to the zero point given in Fig. 2.
was optimized by the $\mathrm{CNDO} / 2$ method in three steps. First, the value of $\Theta_{\mathrm{N}}$ corresponding to a minimum total energy at the original value of the bond length was found ${ }^{\text {( }} 21 \cdot 4^{\circ}$; Fig. 2). Using this value of $\Theta_{\mathrm{N}}$, the $\mathrm{N}-\mathrm{H}$ bond length was optimized ( 0.1065 nm ; Fig. 3). Then a second energy minimization with respect to the angle $\Theta_{\mathrm{N}}$ was performed (Fig. 2). The overall energy minimum was found at $\Theta_{\mathrm{N}}=26.4^{\circ}$; this value satisfies Eq. (2). Optimizing the position of the hydrogen atom, a total energy decrease of approximately $29 \mathrm{kcal} \mathrm{mol}^{-1}$ was reached. Most of this change results from lengthening of the $\mathrm{N}-\mathrm{H}$ bond (approximately $28.5 \mathrm{kcal} \mathrm{mol}^{-1}$ ) while variations of $\Theta_{\mathrm{N}}$ are much less critical. (The energy increase in the range of $\Theta_{\mathrm{N}} 15-35^{\circ}$ is less than $0.5 \mathrm{kcal} \mathrm{mol}^{-1}$ ). Then, the preferred geometry of the amide group obtained in this way was utilized in the CNDO/S calculation of the UV and CD spectra (Table II).

Owing to the possibility of small variations of $\Theta_{N}$, and since an analogous motion of the hydrogen atoms was demonstrated in the formamide molecule on the basis of an analysis of microwave spectra ${ }^{24}$, it appears useful to study a relation connecting the spectral parameters of both the $n-\pi^{*}$ and $\pi-\pi^{*}$ transitions of the lactam $(-)-I$ with the value of this angle. The optical rotatory strengths of the both transitions (Fig. 5) cross a nodal plane in the range close to the planar arrangement of the bonds attached to the nitrogen atom $\left(\Theta_{\mathrm{N}} \sim 0^{\circ}\right)$. The shortest wavelength of the $n-\pi^{*}$ transition is also computed in close proximity to this plane (Fig. 6). On the other hand, the computed position of the $\pi-\pi^{*}$ transition exhibits only a monotonous hypso-


Fig. 4
Optimized Geometry of the Amide Group of Lactam (-)-I

For clarity the dihedral angles were slightly enlarged.


Fig. 5
Optical Rotatory Strengths of the $n-\pi^{*}$ and $\pi-\pi^{*}$ Transitions of Lactam (-)-I Computed as Functions of the Pyramidal Arrangement on the Amide Nitrogen (the value $\Theta_{\mathrm{N}}$ )
chromic shift with an increase in $\Theta_{\mathrm{N}}$. If we compare changes in the static charge distribution on the atoms constituting the amide group in the ground state, and in the first electronically excited singlet state relative to the value of $\Theta_{\mathrm{N}}$ (Fig. 7 ), we observe remarkably anomalous values of charge on nitrogen, oxygen and carbon atoms in the range of the nodal plane mentioned above.

## Table II

CNDO/S Calculation of the Optical Rotatory Strength of Lactam ( - )-I. Optimized Molecular Geometry was Used

| Quantity | $n-\pi^{*}$ transition | $\pi-\pi^{*}$ transition |
| :--- | :---: | :---: |
| $\lambda_{\text {exp. }}{ }^{a}, \mathrm{~nm}$ | 232.0 | 197.0 |
| $\lambda_{\text {calc. }}, \mathrm{nm}$ | 289.0 | 163.8 |
| $f_{\text {exp. }}{ }^{\text {a }}$ | 0.0056 | 0.0345 |
| $f_{\text {calc. }}$ | 0.0082 | 0.1316 |
| $R_{\text {exp. }} .{ }^{a} \cdot 10^{39}, \mathrm{cgs}$ | -2.51 | 2.07 |
| $R_{\text {calc. }} .10^{39}, \mathrm{cgs}$ | -2.51 | 1.21 |

[^0]Fig. 6
The Shifts of the $n-\pi^{*}$ and $\pi-\pi^{*}$ Transitions in the Molecule of ( - )-I Computed Relative to $\Theta_{\mathrm{N}}(\mathrm{CNDO} / \mathrm{S})$


## DISCUSSION

The polycyclic structure of lactam I gives this molecule a high degree of rigidity. Considering this property, as well as the bigh precision of X -ray analysis ${ }^{14}$, the possibility that some skeletal atoms deviate significantly on change of state of the molecule from crystals to solution would appear improbable. The same precision is reached in determining the position of the oxygen atom of the amide group. In addition, the pyramidal character exhibited at the carbonyl carbon atom is usually negligible according to Ramachandran and coworkers ${ }^{8-10}$. Therefore, optimization of the molecular geometry with respect only to the less accurately known positions of the hydrogen atoms is justified. Chiroptical properties of the lactam in question can be significantly influenced only by the hydrogen bonded to the nitrogen of the amide group. For these reasons, optimization was limited to a search for the most advantageous position of this atom.

On the contrary to the situation in the crystalline state, the value of angle $\Theta_{N}$ found by optimization conforms well to relation (2). Hence, from the geometrical point of view, a regular pyramidal arrangement on the nitrogen atom (i.e. both substituents on N deviate from the $\mathrm{O}-\mathrm{C}-\mathrm{N}$ plane by approximately the same angle)


Fig. 7
Alterations of the Charge Distribution in the Amide Group of Lactam (-)-I Related to the Developing Pyramidal Arrangement of Bond Attached to the Nitrogen Atom ( $-\bullet$ - $)$ Ground State Charge Distribution; (O-O-O) First Excited Singlet State Charge Distribution

The values given are in atomic units.
is found. An analogous situation was detected ${ }^{25}$ by computation also in the case of 2-acetamido-1,3-propandiol, studied in connexion with the chiroptical properties of acetylated 3-amino-3-deoxyhexoses. However, the structure of 2-acetamido-1,3--propandiol is flexible and has a trans-amide arrangement. Therefore this compound represents a case similar to earlier investigated molecules ${ }^{8-10}$.

This study of lactam $I$ shows that the trend of forming a regular pyramid on the nitrogen atom has a more general validity if the presented quantum chemical calculations are accepted as reliable. The cause of this preference for the regular pyramid may follow from the tendency to maximize the overlap between the orbitals of the carbonyl $\pi$ system and the lone electron pair on nitrogen, even in the case of the nonplanar amide group.

The values of optical rotatory strengths related to $n-\pi^{*}$ and $\pi-\pi^{*}$ transitions, which were computed for the optimized molecular geometry (Table II), agree well with the experimental results. (The computed values were compared with data measured in cyclohexane solution ${ }^{11}$, since the approximation of the isolated molecule is best fulfilled in a non-polar solvent). The sole quantities which remain inaccurate are the positions of both transitions. This inaccuracy involves unknown positions of the $0-0$ components of both bands and perhaps the paramerization used. Similar disagreements of experimental and theoretical positions of electronic transitions within the amide group have been reported elsewhere ${ }^{4-6}$.

The computed oscillator strength of the $n-\pi^{*}$ transition is close to the experimental value (Table II). The less reliable result concerning the $\pi-\pi^{*}$ transition (the computed value was approximately four times the experimental) can be explained in that only part of this absorption band lies in the accessible spectral region.
If we suppose that at low temperature the hydrogen atom is fixed in its equilibrium position, then according to a calculated relation between rotatory strength and $\Theta_{\mathrm{N}}$ the Cotton effects connected with both bands under study should exhibit a hyperchromic effect on lowering the temperature. Such an effect was actually found ${ }^{11}$. The observed hypsochromic shift of the both bands can be explained by a simultaneous action of solvent and the stabilization of the hydrogen atom position. Crossing the nodal plane, the optical rotatory strength of the $n-\pi^{*}$ transition undergoes a remarkable change and rapidly reaches a relatively high absolute value (Fig. 5). With the exception of the narrow range of $\Theta_{\mathrm{N}}$ close to the nodal plane, these values are always greater than those corresponding to the $\pi-\pi^{*}$ transition. This conclusion agrees with the previous observation ${ }^{25,26}$ that non-planarity of the amide group is manifest by an increase in the intensity of the Cotton effect corresponding to the $n-\pi^{*}$ transition. The remarkable change of the $n-\pi^{*}$ rotatory strength in the proximity of the nodal plane is probably connected with a qualitative alteration of the character of this transition, which takes place when the arrangement of bonds on nitrogen becomes pyramidal. This suggestion follows the analysis of the static
charge distribution in the ground state and in the first electronically excited singlet state (Fig. 7). If hybridization of the amide nitrogen is $s p^{2}\left(\Theta_{\mathrm{N}}=0^{\circ}\right)$ then the charge on this atom is practically equal in both the ground and first excited states and consequently the corresponding lone electron pair does not participate in the excitation process. On the other hand, a decrease of the negative charge on the oxygen atom in the course of this excitation indicates that with a planar arrangement of bonds on the amide nitrogen, the $n-\pi^{*}$ transition can be described as a transition of an electron from the oxygen $n$ orbital. However, the pyramidal arrangement on nitrogen substantially alters this situation. In the latter case, the excitation process is accompanied by an appreciable decrease of the negative charge on nitrogen and the originally large charge alteration on the oxygen atom is diminished. Hence, it may be argued that at non-zero values of $\Theta_{\mathrm{N}}$, characterizing a pyramidal arrangement on the nitrogen atom the $n-\pi^{*}$ transition arises to a great extent from the nitrogen $n$ orbital.

Thus, the inherent chirality of the amide group in the lactam $(-)-I$ seems to have its primary source in the pyramidal arrangement on the nitrogen atom. As a consequence of this arrangement the $n-\pi^{*}$ transition is qualitatively altered and an experimentally detectable increase of the corresponding rotatory strength occurs. It is probable that the signs of both Cotton effects are then determined by a respective orientation of the lone electron pair on nitrogen and of the $\mathrm{C}=\mathrm{O}$ bond of the amide group.

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[^0]:    ${ }^{a}$ See note under the Table I; ${ }^{b}$ oscillator strength measured in 3-methylpentane.

